Trends in moments of climatic indices

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[1]   We propose a new technique to analyze trends in moments of the statistical distribution of climatic indices. A standard approach (linear regression, polynomial fit, or least squares fit to a specific function) is first used to evaluate the trend in the expected value of an observed climatic index. The innovation here is that after the trend in the expected value is subtracted from the observed time series, we calculate the time series of the squares, cubes, fourth powers, and any other combination of the residuals (anomalies). Then we apply the same standard trend analysis technique to the time series of these new variables. This technique can be used to determine whether the observed climate is getting more or less variable. The observed 1901–2000 New York City sea level variations, U.S. annual average precipitation, U.S. annual averages of the Modified Palmer Drought Severity Index, All-India Monsoon Rainfall Index, and Southern Oscillation Index are used to illustrate the technique. There are no significant trends in variability of any of these climatic indices for the past 100 years.   INDEX TERMS: 1694 Global Change: Instruments and Techniques, 3309 Meteorology and Atmospheric Dynamics Climatology 1620

1.   Introduction

[2]   Historical climatic data are mostly monthly or annual averages of meteorological variables. Many generations of climatologists have been analyzing such data in the past and work with such data now. A technique for studying the trends in the expected values (normals) of climatic parameters has been well developed and is based on least squares fitting of observed data to linear or polynomial functions [*Anderson*, 1971; *Polyak*, 1979]. But a much more difficult question to answer is whether the climate is getting more or less variable. Such a question is really a question about the trends in the variance and higher moments of the statistical distribution of climatic variables. Here we show how such trends can be studied.

2.   Trends in Moments of the Statistical Distribution of Annual or Monthly Averages

[3]   Let $y(t)$, $t = t_1, t_2, t_3, \ldots, t_n$ be a time series of n observed annual (or monthly for a specific month) averages of a climatic variable y, and t is the year number. Let us suppose that the expected value Y_1, the first moment of the statistical distribution of variable y, has a linear trend for the time interval (t_1, t_n):

$$Y_1(t) = E[y(t)] = a_1 + b_1 t. \qquad (1)$$

The operator $E(y)$ is the expected value of y. The parameters a_1 and b_1 can be estimated using a least squares fit or linear regression. This approach is more or less common in modern climatology, and the statistical significance of the trend can be evaluated [*Karol et al.*, 1976; *Polyak*, 1979; *Vinnikov*, 1986; *Press et al.*, 1986]. The shape of the climatic trend may not be a linear function of time, however. We are looking here for linear trends in climatic records for simplicity only.

[4]   The residuals,

$$y'(t) = y(t) - Y_1(t) = y(t) - a_1 - b_1 t, \qquad (2)$$

have no linear trend in expected value. However, we can expect that the second moment of the statistical distribution Y_2 of these residuals has a linear trend for the time interval (t_1, t_n):

$$Y_2 = Y_2(t) = E\left[y'(t)^2\right] = a_2 + b_2 t. \qquad (3)$$

Analogous hypotheses can be formulated for the third (Y_3) and the fourth (Y_4) moments of the statistical distribution:

$$Y_3 = Y_3(t) = E\left[y'(t)^3\right] = a_3 + b_3 t; \qquad (4)$$

$$Y_4 = Y_4(t) = E\left[y'(t)^4\right] = a_4 + b_4 t. \qquad (5)$$

The parameters a_2, a_3, a_4, b_2, b_3, and b_4 can be estimated in the same way as the parameters a_1 and b_1 and the hypotheses (3–5) can be accepted or rejected in the same way as for hypothesis (1). We can also look for trends in higher moments and in lag-covariance/correlation or cross-covariance/correlation of variables. Here we illustrate these ideas with a few simple illustrations.

3.   Application of the Technique

[5]   As an example of this technique, we use it to study changes in the mean, variance, and higher moments of several observed time series. We use observations for the period of 100 years, 1901–2000. This was a century with an observed global warming trend (~ 0.7 K/100 yr) in surface air temperature [*Jones et al.*, 1999]. We are looking for related trends that may exist in regional climatic variation for the same period. We estimated the parameters in models (1, 3–5) for five climatic indices that are traditionally in use to monitor large-scale climatic variations:

[6]   *Sea level at New York.* The observed annual averages of sea level at tidal station New York were retrieved electronically from the Permanent Service for Mean Sea Level [*Woodworth*, 1991]. We use a record from a single station, because no official data for global sea level variation exist. Figure 1 shows the estimated trends for each of the first four moments of this variable. We use estimates of the root mean square errors ($rmse_{bi}$) of the coefficients b_i, $i = 1$, 2, 3, 4 based on the assumption that the variations in the time series

$y(t)$ are independent. These estimates are given at the top of each panel in each of the figures. We used them and a two-sided Student t-test to verify the hypothesis that the estimates of the b_i are statistically significantly distinct from zero. The trend in the first moment, a sea level rise of about 30 cm/100 yr, is statistically significant at more than the conventional 95%-level, which is used in this paper as the threshold for a decision on the significance of trend estimates. The trends in variance and other moments are not large enough compared to their sampling errors to be significant.

[7] *U.S. precipitation*. The U.S. average annual precipitation time series has been calculated by the National Climatic Data Center from observations of the U.S. Historical Climatology Network [*NCDC*, 1994]. This index is traditionally in use for monitoring of recent climate change. Results of the trend analysis (Figure 2) show a statistically significant upward trend of 5.8 cm/100 yr in the first moment, but no significant trends in the second, third or fourth moments of this index.

[8] *Palmer drought index*. The U.S. Modified Palmer Drought Severity Index is a widely-used measure of agricultural soil moisture [*Heddinghause and Sabol*, 1991; *NCDC*, 1994]. Trend analysis of the annual averages of this index (Figure 3) shows no significant trends in any of the first four statistical moments of this index.

[9] *Indian summer monsoon precipitation*. The All-India monsoon rainfall index [*Parthasarathy et al.*, 1995], the area-weighted total of June, July, August, and September precipitation for 29 districts covering India, is the most commonly used index of monsoon long-term variability. None of the trends in the first four statistical moments (Figure 4) are statistically significant.

[10] *El Niño*. The Southern Oscillation Index [*Ropelewski and Jones*, 1987; *Können et al.*, 1998] also does not have statistically significant trends in the first four moments (Figure 5). This index,
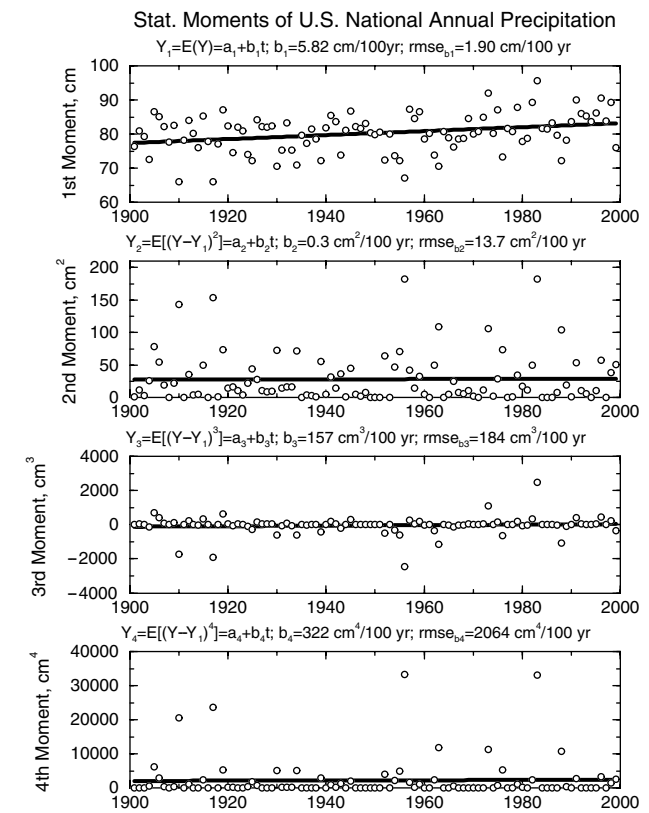

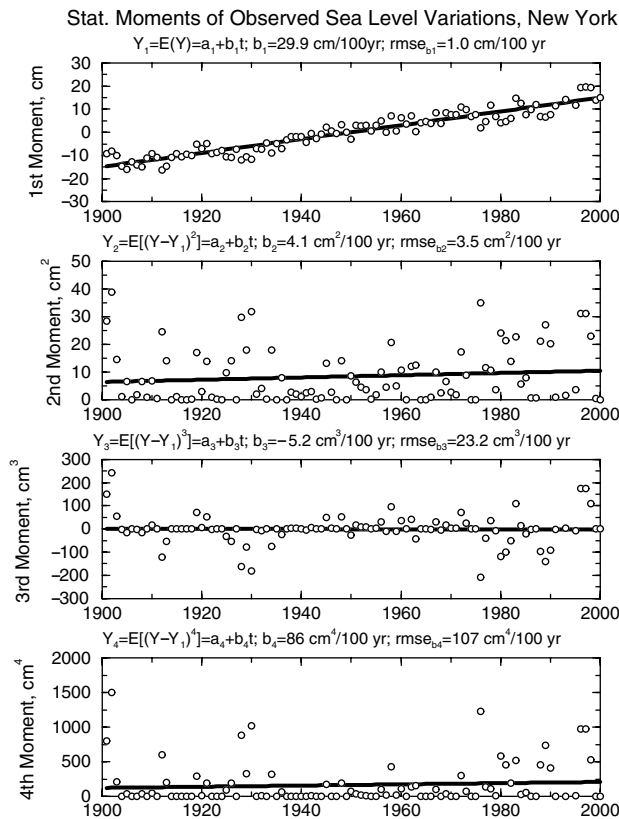**Figure 2.** The same as in Figure 1, But for U.S. National Annual Precipitation.

Figure 1. Estimates of linear trends in the first four moments of the statistical distribution of sea level at the New York tidal station. Circles-annual data. Solid lines-linear trends.
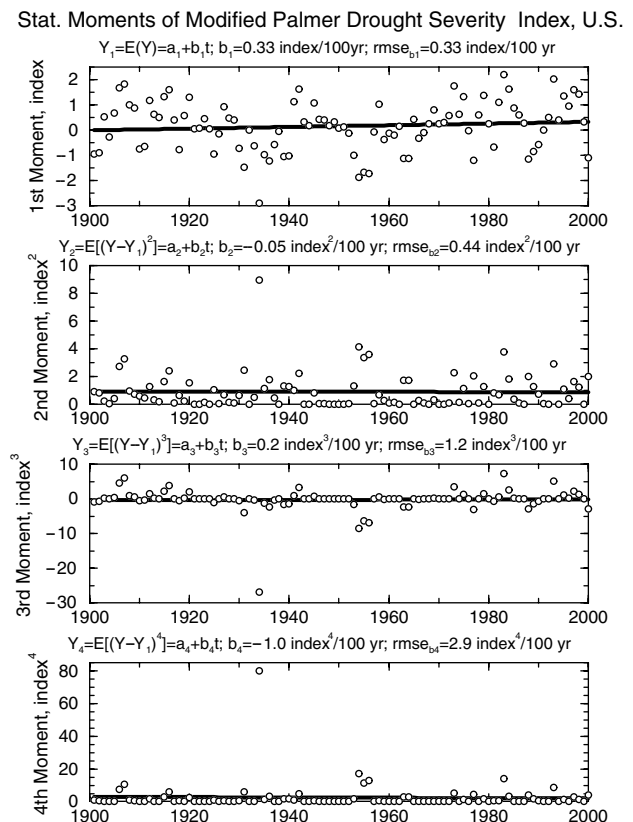

Figure 3. The same as in Figure 1, But for Modified Palmer Drought Severity Index; U.S., annual averaged.
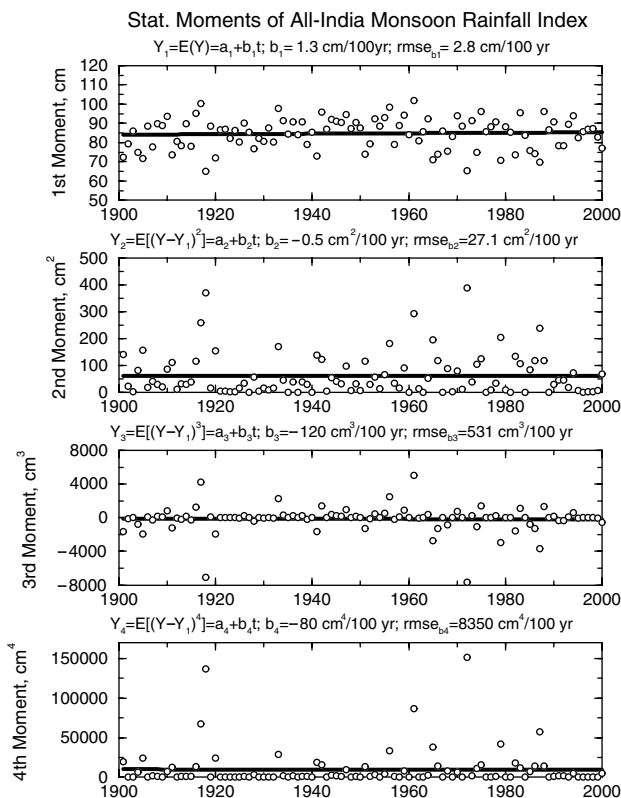
calculated from sea level pressure, reflects changes in atmospheric circulation, but not a global warming signal in the tropical ocean temperature.

4. Discussion

[11] The trend analysis presented above shows that the observed 20th Century global warming trend has been accompanied by trends in the first statistical moments of some, but not all, climatic indices. We found no significant trends in the second, third, and fourth moments of the statistical distributions of five selected climatic indices. Contrary to previous suggestions we found no change in climate variability for any of these indices for the past 100 years.
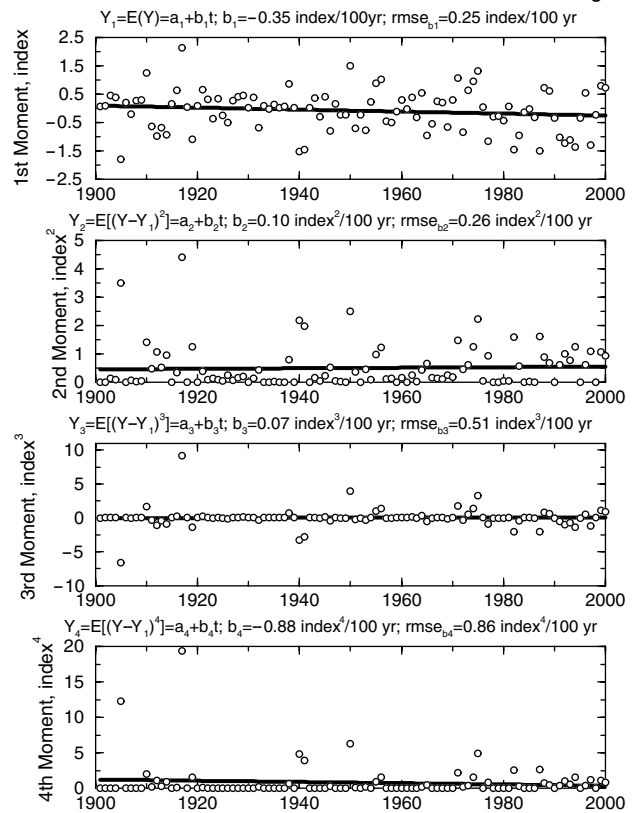
[12] These illustrations demonstrate how the proposed technique works. The innovation here is that after the trend in the expected value is subtracted from the observed time series, we can calculate time series of squares, cubes, fourth powers, and any other combinations of the residuals (anomalies). Then we apply the same standard trend analysis technique to the time series of these new variables. Trend estimates for each specific month of a year give us information about the seasonality of trends in moments of the statistical distribution of climatic variables. But much more information about the seasonality of trends can be received if we use daily or hourly observations. Statistical techniques developed to analyze the seasonal and diurnal cycles in climatic trends will be discussed in two other papers.

[13] The most complicated part of trend analysis is choosing a basis function. Additional information should be used, if possible, to decide whether the trend is linear, quadratic, exponential, or some other shape. Detrending always removes trend-like components of a random process. The more parameters in the trend function, the more likely detrending is to affect trend estimates of

Figure 5. The same as in Figure 1, but for annual averages of the Southern Oscillation Index.

the higher order statistics. To minimize this effect, 1) the number of parameters in the trend function should be much less than the number of independent observations in the time series, and 2) each additional parameter should substantially decrease the variance of the residuals. We performed experiments with polynomial functions for trends for the examples presented in this paper, and found that none of the conclusions of this analysis depend on the order of the polynomial trend.

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Figure 4. The same as in Figure 1, but for All-India Monsoon Rainfall Index.

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